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XXXVI. *Some New Theorems for computing the Areas of certain Curve Lines: By Mr.* John Landen, *F.R.S.*

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THE learned editor of Mr. Cotes's *Harmonia Menſurarum* firſt gave us, in that book, the celebrated theorems for computing the areas of the curves whoſe ordinates are expreſſed by $\frac{x^p}{a^n+x^n}$, $\frac{x^p}{\overline{a^n+x^n}\times\overline{e^n+x^n}}$, or $\frac{x^p}{a^{2n}+2ca^nx^n+x^{2n}}$; and ſeveral other writers have ſince done the like. Which theorems conſiſt of many terms, being obtained by previouſly reſolving the expreſſion for the ordinate, into others of a more ſimple form. Now I have found, that the *whole* area of every ſuch curve (when finite) may be aſſigned by theorems remarkably conciſe, without the trouble of reſolving the expreſſion for the ordinate as aforeſaid: and, as in the reſolution of problems, the whole area of a curve is more commonly wanted than a part thereof; and as theſe new theorems enable us to compute ſuch whole areas as above-mentioned, or the whole fluents of $\frac{x^p\dot{x}}{a^n+x^n}$, $\frac{x^p\dot{x}}{\overline{a^n+x^n}\times\overline{e^n+x^n}}$, and $\frac{x^p\dot{x}}{a^{2n}+2ca^nx^n+x^{2n}}$, with

admirable facility; I do myſelf the honour of communicating them to the Royal Society, preſuming they may be thought worthy to be publiſhed in the Philoſophical Tranſactions.

THEOREM I.

m being any poſitive integer or fraction, and *n* any ſuch integer or fraction, greater than *m*; the *whole* area of the curve, whoſe abſciſſa is x, and ordinate $\frac{x^{m-1}}{a^n+x^n}$ is $=\frac{a^{m-n}}{fn}\times A$.

THEOREM II.

m and *n* being as before-mentioned, the *whole* area of the curve, whoſe abſciſſa is x, and ordinate $\frac{x^{n\pm m-1}}{\overline{a^n+x^n}\times\overline{e^n+x^n}}$ is $=\pm\frac{a^{\pm m}-e^{\pm m}}{a^n-e^n}\times\frac{A}{fn}$.

Note. When e is $=a$, the expreſſion for the area becomes $=\frac{ma^{\pm m-n}}{fn^2}\times A$.

THEOREM III.

m and *n* being as in the preceding theorems, the *whole* area of the curve, whoſe abſciſſa is x, and ordinate $\frac{x^{n\pm m-1}}{a^{2n}+2ca^nx^n+x^{2n}}$ is $=\frac{g\,a^{\pm m-n}}{bfn}\times A$.

Note. If m be $=o$, the area will be $=\frac{a^{-n}B}{bn}$.

In theſe theorems,

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A denotes the ſemi-periphery of the circle, whoſe radius is 1;

B an arc of the ſame circle, whoſe coſine is c, and ſine b;

f the ſine of the arc $\frac{m}{n} \times A$;

g the ſine of the arc $\frac{m}{n} \times B$.

Concerning the inveſtigation of theſe theorems, it is ſufficient to ſay, they are directly obtained by the help of my new method of comparing curvilineal areas, inſerted in the Philoſ. Tranſact. for the year 1768.

It is obvious, that, by means of the above theorems, we may very readily compute the *whole* areas (when finite) of the curves, whoſe ordinates are $\frac{\dot{x}p}{p+qx^n+rx^{2n}+x^{3n}}$, and $\frac{\dot{x}p}{p+qx^n+rx^{2n}+sx^{3n}+x^{4n}}$, &c. ſeeing theſe expreſſions may be eaſily transformed into others ſimilar to thoſe already conſidered.